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Roll Resonance Control of Angle of Attack for Re-entry Vehicle Drag Modulation

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Slender ballistic re-entry vehicles are highly susceptible to roll-rate changes caused by small asymmetries in mass and configuration. One particular combination of asymmetries, consisting of a body-fixed trim angle of attack and a small radial center-of-gravity (c.g.) offset, can cause a rapid spinup into resonance, with a large amplification in trim angle of attack and a concomitant increase in drag. By controlling the magnitude and direction of the c.g. offset with a moving-mass roll control system, it is possible to control the roll rate near resonance, and to limit the angle of attack and drag response to a controlled value. A feedback law is derived and the control system is demonstrated with a digital computer simulation of the equations of rotational motion.

Nomenclature

A_N = normal acceleration
A_{N_c} = normal acceleration command
a,b,c = roots of Eq. (15)
c = radial c.g. offset
C_A = axial force coefficient
C_D = drag coefficient
C_L = lift coefficient
C_{m_q} = pitch damping derivative
$C^*_{m_q}$ = $-C_{m_q}qSd^2/2Iu$
C_N = normal force coefficient
C_{N_α} = normal force derivative
$C^*_{N_\alpha}$ = $C_{N_\alpha}qS/mu$
d = aerodynamic reference (base) diameter
g = acceleration due to gravity
h = altitude
h_0 = altitude at recovery initiation
I = pitch or yaw moment of inertia
I_x = roll moment of inertia
K = feedback gain = $\omega\tau K_2/2$
K_1,K_2,K_3 = feedback gains
m = vehicle mass
p = roll rate
p_0 = roll rate at recovery initiation
q = dynamic pressure
R_E = Earth radius
s = Laplace transform variable
S = aerodynamic reference (base) area
t = time
u = velocity
u_0 = velocity at recovery initiation
x_{st} = static margin
β = $\sin^{-1}\lambda$
γ = path angle
γ_0 = path angle at recovery initiation
ϵ = roll angle = $\pi/2-\phi$
η = $C_{N_\alpha}qS/I_x$
η_1 = $C_N(\theta)qS/I_x$
θ = angle of attack
θ_c = command value of angle of attack
$\dot\theta$ = pitch rate in wind coordinates
λ = $\tan\sigma/\tan\theta$
μ = I_x/I
ν = pitch or yaw damping parameter = $C^*_{m_q}+C^*_{N_\alpha}$
ρ = atmospheric density
σ = cone half angle
τ = trim angle of attack asymmetry
ϕ = roll angle relative to wind
ϕ_0 = meridian angle between trim asymmetry and c.g. offset (Fig. 2)
ψ = precession angle
$\dot\psi$ = precession rate
ω = undamped natural pitch frequency
Ω = controller frequency = $(\eta\theta c)^{1/2}$

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Introduction

IT IS well known that a slender, high-performance ballistic re-entry vehicle is susceptible to roll resonance caused by small asymmetries in mass and configuration.[1-4] The vehicle can exhibit large roll-rate excursions because of its low roll moment of inertia and the extreme aerodynamic pressures during atmospheric entry. The vehicle is most susceptible to a body-fixed trim asymmetry with an orthogonal component of radial c.g. offset. Lift caused by the trim angle of attack acts on the c.g. offset moment arm to produce a roll torque that can spin the vehicle rapidly into resonance. A trim angle-of-attack asymmetry on the order of 1 deg or less in conjunction with a radial c.g. offset on the order of tens of thousandths of an inch can cause large roll-rate excursions sufficient to spin the vehicle into resonance. The trim angle of attack can be amplified in resonance by a factor of 10 to 15 or more, depending on the altitude at which resonance is encountered. As a preventive measure, vehicles must be accurately balanced to minimize radial displacement of the center of gravity from the aerodynamic axis of symmetry. In some cases, roll control is also required.

Described here is a method of utilizing roll resonance in a controlled manner for drag modulation. Large angle-of-attack-induced drag can be used to decelerate a test vehicle for recovery,[5] or drag modulation can be used to compensate for drag uncertainty in order to control range errors.[6] The control system consists of a moving-mass roll control, strapdown motion sensors, and a built-in trim angle-of-attack asymmetry. Moving-mass roll control systems have been developed that are suitable for the proposed application. During roll resonance, the roll rate is approximately equal to the undamped natural pitch frequency, which can become quite large for a ballistic re-entry vehicle near peak dynamic pressure. However, the frequency at which the roll rate oscillates about the critical frequency, which determines the response frequency of the moving-mass control system, is considerably lower than the pitch frequency. This feature of the control system as well as the relatively small mass required for c.g. control greatly simplifies the practical im-

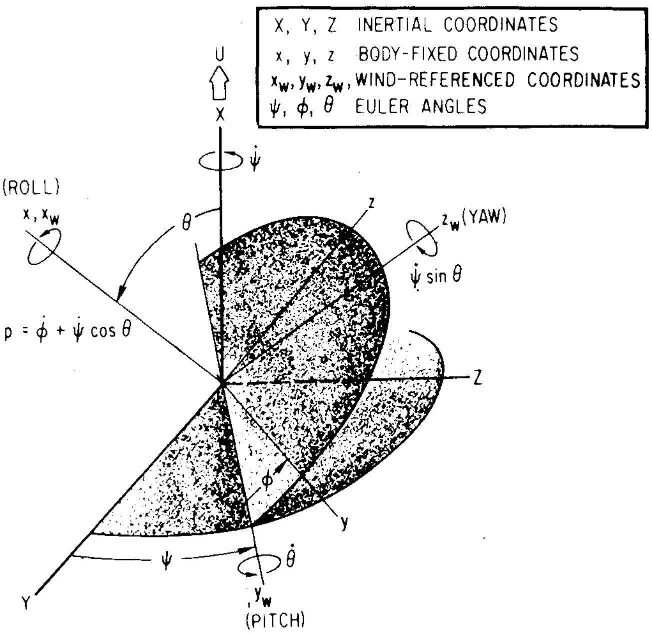

Fig. 1 Wind-referenced coordinates.

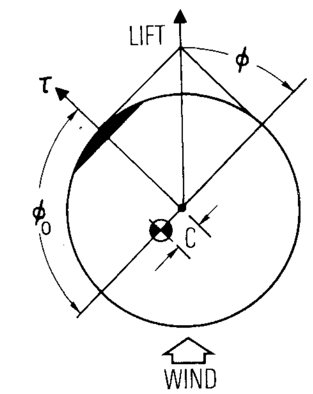

Fig. 2 Asymmetries and roll orientation.

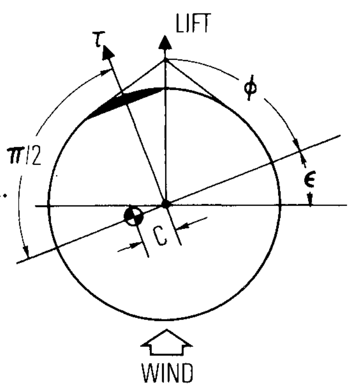

Fig. 3 Superresonant condition.

plementation of the system. An ideal control law is postulated from a linear approximation to the moment equations of motion near resonance. Feedback gains are determined from a root locus analysis to give a stable response to the linear system, with little consideration given to optimizing performance. The control system as applied to recovery is demonstrated with a digital-computer simulation of the nonlinear moment equations of motion.

Control Analysis

The small-angle equations of rotational motion in terms of classical Euler angles or wind-referenced coordinates (Fig. 1) can be written[3,4]

$$\ddot{\theta} + (\omega^2 - \dot{\psi}^2)\theta + \nu\dot{\theta} = \omega^2\tau\cos(\phi + \phi_0) \tag{1}$$

$$\frac{d}{dt}(\dot{\psi}\theta) + \dot{\theta}\dot{\psi} + \nu\dot{\psi}\theta = \omega^2\tau\sin(\phi + \phi_0) \tag{2}$$

$$\dot{p} = -\eta\theta c\sin\phi \tag{3}$$

in which τ is a body-fixed, nonrolling trim angle of attack, oriented at an angle ϕ_0 with respect to the plane of a radial c.g. offset c (Fig. 2). The roll angle ϕ is a measure of the orientation of the plane of c.g. offset with respect to the wind plane. We will consider the case in which the trim is orthogonal to the plane of c.g. offset ($\phi_0 = 90$ deg), which will cause a rapid spinup into resonance. We assume, a priori, the quasisteady condition

$$\dot{\psi} \approx \omega = \text{const} \tag{4}$$

which, with the small-angle definition of p

$$p = \dot{\phi} + \dot{\psi} \tag{5}$$

gives, for Eqs. (2) and (3),

$$\dot{\theta} + (\nu/2)\theta = (\omega\tau/2)\cos\phi \tag{6}$$

$$\ddot{\phi} + \eta\theta c\sin\phi = 0 \tag{7}$$

We further assume that the vehicle spins through resonance so that the body-fixed trim plane is oriented nominally 180 deg from its nonrolling orientation, $\phi = -90$ deg (Fig. 3), and define a new roll angle ϵ

$$\epsilon = \pi/2 - \phi \tag{8}$$

If we assume that ϵ is a small angle, Eqs. (6) and (7) can be written

$$\dot{\theta} + (\nu/2)\theta = (\omega\tau/2)\epsilon \tag{9}$$

$$\ddot{\epsilon} - \eta\theta c = 0 \tag{10}$$

As the vehicle is driven toward steady resonance, which is a stable trim point with a fixed trim and orthogonal c.g. offset, it is desired that the controller limit the trim angle-of-attack amplification or normal acceleration to a controlled value by adjusting the magnitude and direction of the c.g. offset. An ideal control law postulated to linearize Eqs. (9) and (10) has the form

$$c = -\frac{K_1\dot{\epsilon} + K_2(\theta - \theta_c) + K_3\dot{\theta}}{\eta\theta} \tag{11}$$

in which θ_c is a command value of angle of attack (or normal acceleration). To implement Eq. (11), $\dot{\epsilon}$ is measured (the rotation rate of the lift vector relative to body-fixed coordinates) and the numerator is divided by θ. This can be accomplished with two body-mounted orthogonal accelerometers in conjunction with a digital microcomputer that would differentiate the lift vector orientation angle $\tan^{-1} A_y/A_z$, for example, and divide the numerator expression by the resultant normal acceleration $(A_y^2 + A_z^2)^{1/2}$. Equation (11) substituted in Eq. (10) gives

$$\ddot{\epsilon} + K_1\dot{\epsilon} + K_2(\theta - \theta_c) + K_3\dot{\theta} = 0 \tag{12}$$

which, with Eq. (9), yields two coupled control equations in θ and ϵ. We can solve for θ from Eqs. (9) and (12) by the use of

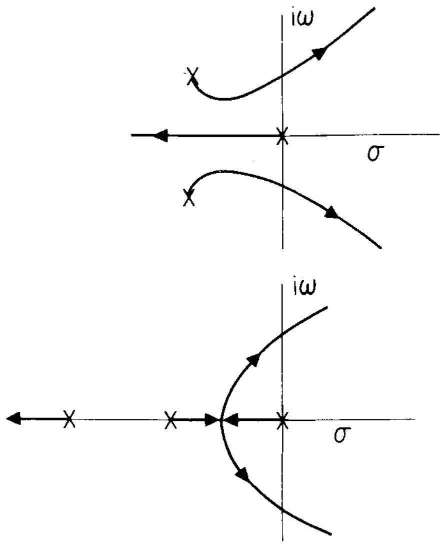

Fig. 4 Root loci.

Laplace transforms, which gives

$$\left[s^3 + \left(K_1 + \frac{\nu}{2}\right)s^2 + \left(\frac{\nu}{2}K_1 + \frac{\omega\tau}{2}K_3\right)s + \frac{\omega\tau}{2}K_2\right]\theta(s)$$

$$= \frac{\omega\tau}{2}K_2\theta_c(s) \tag{13}$$

Consider the response to a step command $\theta_c(t) = \theta_c$. Equation (13) can then be written

$$\theta(s) = \frac{K\theta_c}{s(s+a)(s+b)(s+c)} \tag{14}$$

where

$$(s+a)(s+b)(s+c) = s^3 + \left(K_1 + \frac{\nu}{2}\right)s^2 + \left(\frac{\nu}{2}K_1 + \frac{\omega\tau}{2}K_3\right)s + K \tag{15}$$

and $K = \omega\tau K_2/2$. We can obtain a, b, and c from a root locus in which the transfer function G is

$$G = \frac{K}{s\left[s^2 + \left(K_1 + \frac{\nu}{2}\right)s + \frac{\nu}{2}K_1 + \frac{\omega\tau}{2}K_3\right]} \tag{16}$$

Two possible root loci are shown in Fig. 4, depending on the coefficients of s in Eq. (16). For a stable solution, either three negative real roots or one negative real and two complex conjugate roots are possible. The inverse Laplace transform of Eq. (14) yields the time response to the step command θ_c

$$\frac{\theta(t)}{K\theta_c} = \frac{1}{abc} - \frac{1}{a(a-b)(a-c)}e^{-at} - \frac{1}{b(b-a)(b-c)}e^{-bt} - \frac{1}{c(c-a)(c-b)}e^{-ct} \tag{17}$$

Application to Recovery

The large-angle pitch and yaw equations equivalent to Eqs. (1) and (2) can be written[5]

$$\ddot{\theta} - \dot{\psi}^2\sin\theta\cos\theta + \nu\dot{\theta} = \omega^2\tau\cos(\phi+\phi_0) - C_N(\theta)qSx_{st}/I \tag{18}$$

$$\frac{d}{dt}(\dot{\psi}\sin\theta) + \dot{\theta}\dot{\psi}\cos\theta - \mu p\dot{\theta} + C^*_{m_q}\dot{\psi}\sin\theta - C^*_{N_\alpha}\mu p\theta$$

$$= \omega^2\tau\sin(\phi+\phi_0) + \frac{C^*_{N_\alpha}C_L(\theta)\dot{\psi}\sin\theta\cos\theta}{C_N(\theta)} \tag{19}$$

in which the lift coefficient $C_L(\theta)$ and the normal force coefficient $C_N(\theta)$ can be approximated by the sharp cone Newtonian relations.[7] For $\theta \leq \sigma$

$$C_N(\theta) = \cos^2\sigma\sin^2\theta \tag{20}$$

$$C_A(\theta) = 2\sin^2\sigma + (1 - 3\sin^2\sigma)\sin^2\theta \tag{21}$$

and for $\theta > \sigma$

$$C_N(\theta) = (\cos^2\sigma\sin2\theta)\left[\left(\frac{\beta+\pi/2}{\pi}\right) + \left(\frac{\cos\beta}{3\pi}\right)\left(\lambda + \frac{2}{\lambda}\right)\right] \tag{22}$$

$$C_A(\theta) = \left(\frac{\beta+\pi/2}{\pi}\right)[2\sin^2\sigma + (1-3\sin^2\sigma)\sin^2\theta]$$

$$+ (3/4\pi)\cos\beta\sin2\theta\sin2\sigma \tag{23}$$

where σ = cone half-angle, $\lambda \equiv \tan\sigma/\tan\theta$, $\beta \equiv \sin^{-1}\lambda$, and

$$C_D(\theta) = C_A(\theta)\cos\theta + C_N(\theta)\sin\theta \tag{24}$$

$$C_L(\theta) = C_N(\theta)\cos\theta - C_A(\theta)\sin\theta \tag{25}$$

We can estimate influence of the angle of attack and drag on the trajectory from Eqs. (3), (18), and (19) in conjunction with the trajectory equations[5]

$$\frac{du}{dt} = -\frac{C_D qS}{m} + g\sin\gamma \tag{26}$$

$$\frac{dh}{dt} = -u\sin\gamma \tag{27}$$

$$\frac{d\gamma}{dt} = \left(\frac{g}{u} - \frac{u}{R_E + h}\right)\cos\gamma \tag{28}$$

The moving-mass controller is simulated by the c.g. offset relation given in Eq. (11) in which $\dot{\epsilon} = -\dot{\phi}$ according to Eq. (8). The function of the control system is to limit the normal acceleration to some acceptable value, while providing sufficient angle-of-attack-induced drag to decelerate the vehicle from hypersonic velocity at recovery initiation altitude to a soft landing.

Numerical Example

The equations of motion were solved numerically for a recovery simulation in which a ballistic re-entry vehicle is spun into resonance at an altitude of 15 kft. The moving-mass controller is utilized to control the growth of angle of attack in order to limit the normal acceleration to a prescribed maximum value. The angle of attack is related to normal acceleration A_N according to

$$C_N(\theta) = mgA_N/qS \tag{29}$$

which, for small angles, can be written

$$\theta = \frac{mg}{C_{N_\alpha}qS}A_N = \frac{mgx_{st}}{\omega^2 I}A_N \tag{30}$$

The control law, Eq. (11), expressed in terms of normal acceleration and normal acceleration command A_{N_c} is

$$c=\frac{I_x}{mgA_N}\left[K_I\dot{\phi}+K_2\frac{mgx_{st}}{\omega^2 I}(A_{N_c}-A_N)-K_3\dot{\theta}\right] \tag{31}$$

The control configuration is such that the plane of c.g. control is orthogonal to the plane of trim asymmetry ($\phi_0=90$ deg in Fig. 2). The vehicle having the aerodynamic and mass properties listed below is subjected to a suddenly applied trim angle-of-attack asymmetry $\tau=1.5$ deg with an initial c.g. offset of 0.050 in.:

$C_{m_q}=-6$	$h_0=15$ kft
$C_{N_\alpha}=2$	$u_0=15.75$ kft/s
$d=0.958$ ft	$\gamma_0=26$ deg
$I=3.70$ slug-ft^2	$\sigma=6$ deg
$I_x=0.140$ slug-ft^2	$p_0=50$ rad/s
$m=3.79$ slugs	$A_{N_c}=400\,g$
$S=0.721$ ft^2	
$x_{st}=0.468$ ft	

As the vehicle spins into resonance, the c.g. offset is controlled according to Eq. (31), in which the feedback gains are as follows.

For $t\leq 1.33$ s: $K_I=54$ $K_2=22{,}469$ $K_3=400$

For $t>1.33$ s: $K_I=31.6$ $K_2=845$ $K_3=200$

The gains were estimated from the linear results of Eqs. (14-16) using root locus analysis to give a stable solution, and were changed once to accommodate the large change in dynamic pressure and vehicle characteristic frequencies as the vehicle decelerates. The large angle-of-attack aerodynamics are calculated from the Newtonian approximations, Eqs. (20-25), and the pitch damping derivative, C_{m_q}, is altered as a function of angle of attack, as shown in Fig. 5, to approximate the destabilizing effects of vortex shedding at large incidence described in past experiments.[8,9]

Results of the computer simulation are shown in Figs. 6-8. The roll-rate response relative to the critical roll rate is shown

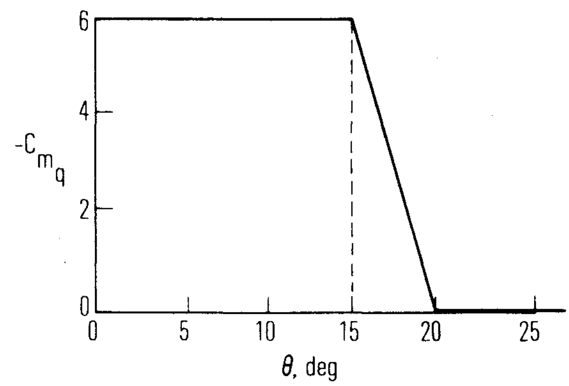

Fig. 5 Idealized approximation to C_{m_q}.

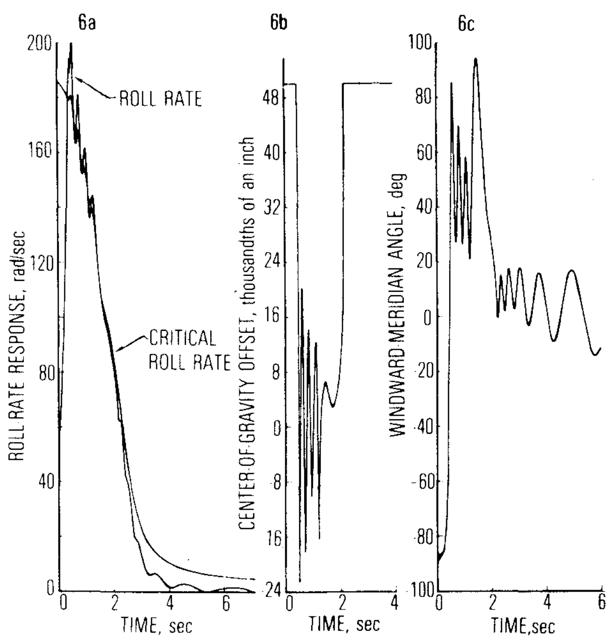

Fig. 6 Roll-rate response, c.g. offset movement, and windward-meridian angle.

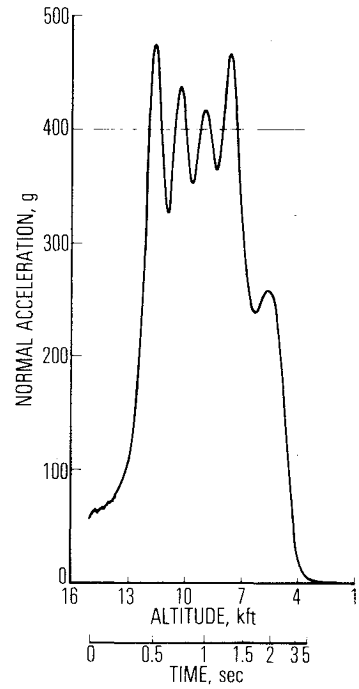

Fig. 7 Acceleration response to 400 g.

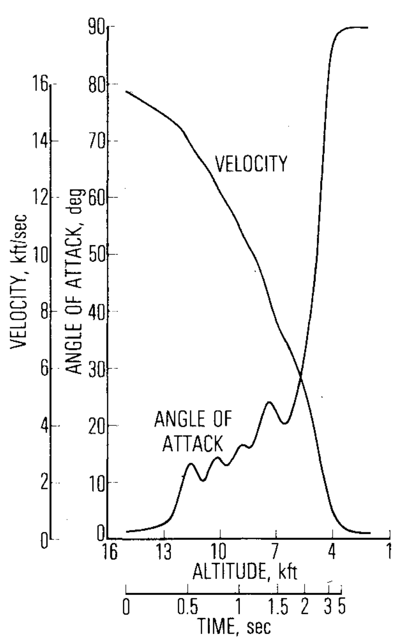

Fig. 8 Behavior of angle of attack and velocity.

in Fig. 6a, the c.g. offset movement is shown in Fig. 6b, and the windward-meridian angle ϕ is shown in Fig. 6c. The normal acceleration response to the command value of 400 g is shown in Fig. 7, and the angle of attack and velocity behavior are shown in Fig. 8. The angle-of-attack growth in resonance with the body-fixed trim asymmetry drives the vehicle into a flat spin at 90-deg angle of attack. A significant feature of the control system response is the relatively low frequency of the c.g. offset or moving-mass oscillations compared with the characteristic pitch frequency of the vehicle. This can be seen from Eq. (7) for the linear approximation to the roll angle oscillations relative to the wind. For small oscillations in ϕ, Eq. (7) represents a harmonic oscillator with natural frequency Ω given by

$$\Omega=(\eta\theta c)^{1/2}\approx\omega[(c/x_{st})(I/I_x)\theta]^{1/2} \tag{32}$$

In the time period around 0.5 to 1 s, from Figs. 6 and 8, $c\approx 0.015$ in., $\theta\approx 12$ deg and $\omega\approx 25$ Hz, which gives for Ω the value 0.12 ω or 3 Hz. This agrees well with the oscillation frequency observed in Fig. 6. The c.g. offset amplitude required for control is approximately ± 0.020 in., from Fig. 6b. This requires a moving-mass throw weight of only 2.5 lb-in., e.g., a 2.5-lb mass with a displacement of ± 1 in. or a 1.25-lb mass with a displacement of ± 2 in., for the 122-lb example vehicle. In view of the relatively low oscillation frequency required for this mass, the energy requirements of the controller are minimal.

Conclusions

The concept of using a moving-mass c.g. controller to limit roll resonance angle-of-attack amplification of a ballistic re-entry vehicle has been demonstrated. An ideal control law postulated from a first-order linear approximation to the nonlinear rotational equations of motion near resonance yields a stable response to an angle-of-attack control system. The first order solution, obtained with little optimization, is an indication of the stability of the resonance lock-in condition driven by a body-fixed trim asymmetry with an orthogonal c.g. offset. A significant feature of the resonant motion is the relatively low frequency of coupled oscillations in angle of attack, roll rate, and roll angle, about the steady-resonance condition. This low-frequency motion determines the response requirements of a moving-mass c.g. controller, and the energy requirements prove to be minimal. The resonance control system has been demonstrated with an application to ballistic re-entry vehicle recovery. The controller can limit the angle of attack and normal load response in resonance, while providing a large angle-of-attack-induced drag for recovery. A digital-computer simulation demonstrates that resonance control can be utilized to decelerate a ballistic re-entry vehicle from high hypersonic velocity at relatively low altitude, to subsonic velocity prior to impact. The vehicle is ultimately driven into a flat spin with enormous drag deceleration, while limiting the normal loads during angle-of-attack buildup to a prescribed value.

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